A Duality Theorem For Graphs, Codes, and Matroids

Thomas Britz

University of New South Wales, Australia

britz@unsw.edu.au

Graphs

Let G be a multigraph with n edges.

Define

k = # of edges in a spanning forest of G

b_i = minimal # of edges in a union of i bonds, none contained in the union of the others

c_j = minimal # of edges in a union of j cycles, none contained in the union of the others.

Set

$U = \{b_1, \ldots, b_k\}$

$V = \{n+1-c_{n-k}, \ldots, n+1-c_1\}$.

Codes

Let C be an $[n,k]$ linear code over some field.

Define for $\mathbf{v} \in C$ and $D \subseteq C$

$\chi(\mathbf{v}) = \{i : v_i \neq 0\}$ $\quad \chi(D) = \bigcup_{\mathbf{v} \in D} \chi(\mathbf{v})$

$w(D) = |\chi(D)|$

$d_i = \min\{w(D)\colon D \text{ an } [n,i] \text{ subcode of } C\}$

$d_j^\perp = \min\{w(D)\colon D \text{ an } [n,j] \text{ subcode of } C^\perp\}$.

Set

$U = \{d_1, \ldots, d_k\}$

$V = \{n+1-d_{n-k}^\perp, \ldots, n+1-d_1^\perp\}$.

Matroids

Let M be a matroid of rank k on n elements.

Define

f_i = maximal size of an i-rank set in M

f_j^* = maximal size of an j-rank set in M^*.

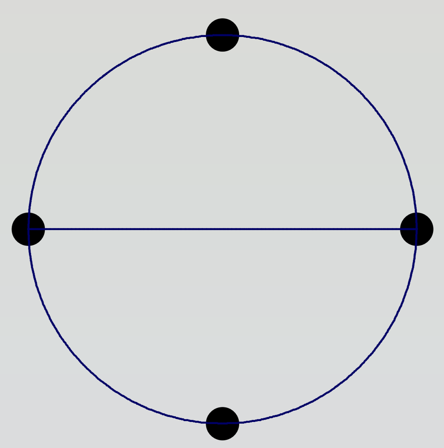
Set

$U = \{f_0+1, \ldots, f_{k-1}+1\}$

$V = \{n-f_{n-k-1}^*, \ldots, n-f_0^*\}$.

Theorem : $U \cup V = \{1, \ldots, n\}$ and $U \cap V = \emptyset$.

Example



$n = 5$ $\quad b_1 = 2$ $\quad U = \{b_1, b_2, b_3\} = \{2,4,5\}$

$k = 3$ $\quad b_2 = 4$ $\quad V = \{5+1-c_2, 5+1-c_1\} = \{1,3\}$

$b_3 = 5$

$c_1 = 3$ $\quad U \cup V = \{1,2,3,4,5\}$ and $U \cap V = \emptyset$

$c_2 = 5$

References

The code and graph theorems first appeared in

V. K. Wei, Generalized Hamming weights for linear codes, *IEEE Trans. Inform. Theory* **37** (1991), 1412–1418.

T. Britz, Higher support matroids, *Discrete Math.* **307** (2007), 2300–2308.